Proximity Point Algorithm for Solving Convex Function Optimization Problems

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Abstract: The development of modern information technology has prompted graphic image-like signal problems to place higher demands on the information processing efficiency of various types of equipment. Such practical application problems can be further transformed into optimization problems for solving convex functions. Therefore, it is especially important to design a more efficient algorithm. Based on this, this paper makes an in-depth study on the neighboring point algorithm for solving convex function optimization problems, hoping to provide reference and reference for solving convex function optimization problems.

1. Research Background

1.1 Literature review

Solving the convex function optimization problem as a special existence optimization problem, more and more researchers began to apply various types of algorithms to the optimization problem of convex functions. Therefore, as an important branch of solving optimization problems in the field of mathematics, this issue has become the focus of attention of all relevant researchers in the academic world. Xu Haiwen and Sun Liming on the convex function optimization problem, combined with the idea of the neighboring point algorithm and the approximate neighboring point algorithm, constructed the expansive descending algorithm direction, and then obtained an accelerated hybrid descent algorithm for a class of convex optimization problems. Afterwards, the Markov inequality, the properties of the convex function and the basic properties of the projection are further used to realize the proof of convergence of the proposed algorithm (xu and sun, 2017).Wang Yiju, for a class of homogeneous polynomial optimization problems encountered in image and signal processing, was first transformed into a target convex function by means of related techniques, and then an algorithm for solving convex function optimization problems was proposed. The method given by the author not only ensures the global convergence of the algorithm under normal circumstances, but also numerical results show that in most cases a global optimal solution of the problem can be obtained (wang, 2010).Wang Shuo et al. proposed a simple original-dual algorithm to solve the minimization problem of the sum of three convex functions. In the proposed original-dual algorithm, the predictive-correction scheme is applied to the dual variable iteration, and the convergence and convergence rate of the algorithm are further analyzed. The final experimental values verify the validity of the original-dual algorithm proposed by the author (wang et al, 2018). Wei Shuhui and Song Guoliang studied the convergence properties of the Broyden family ($\varphi \in [0,1)$) algorithm for non-convex functions with Goldstein line search, and proved the global convergence of the algorithm under certain conditions (wei and song, 2010). Wan Zhong and Feng Dongdong proposed a finely modified Newton method. The method makes full use of the first-order and second-order information of the objective function at the iteration point to further select the appropriate target data search direction. Moreover, the method can establish the global convergence of the algorithm under weaker conditions. Moreover, the final experimental numerical results also show that the algorithm proposed by the author is more efficient than the previous algorithms (wan and feng, 2011).

1.2 Purpose of research

So far, the continuous development of electronic information technology, signal processing,

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image recovery efficiency and other related issues have been the focus of research in related fields. Moreover, these seemingly academic information technology problems can be transformed into solving convex function optimization problems. Therefore, it is extremely important to solve the convex function optimization problem quickly and efficiently. At the same time, related research in recent years also proves that the equivalence relation between the neighboring point algorithm and the sub-gradient is applied to the convex function optimization problem, and further processing the signal image and other problems, usually get better signal results.It can be seen that in the process of solving practical problems, the neighboring point algorithm solves the convex function optimization problem and has strong application value. Moreover, the neighboring point algorithm has great application value for the algorithm research of some micro-calculus and zero-point related problems. However, most of the algorithms involved in solving convex function optimization problems today are weakly convergent. Therefore, it is extremely important to design a strong convergence algorithm based on the neighboring point algorithm. In this context, this paper further proposes a neighboring point algorithm for solving convex function optimization problems. It is expected to supplement the practical application of the near-point algorithm in solving the convex function optimization problem, and provide corresponding reference and reference for subsequent related research.

2. Related Concept Discussion

2.1 Proximity point algorithm

The neighbor algorithm was first proposed by Manite in the 1970s. In general, the neighboring point algorithm is mostly used to solve the problem of lower semi-continuous normal convex function optimization. The neighboring point algorithm is an iterative algorithm, or K nearest neighbor classification algorithm, and in the data mining classification technology, this neighboring point algorithm is usually one of the simplest and most practical methods(wan et al, 2015). The so-called neighbor algorithm (kNN) is the k nearest neighbors, that is, each sample point can be represented by reference to the k nearest neighbors. The key to the neighboring point algorithm is that the target research sample is in the same level in the same space as the k nearest neighbor samples, and the target research sample is also attributed to this level. And the attributes of the samples in this level are available in the target study samples. Therefore, in determining the classification decision process, the neighboring point algorithm usually determines the attribute category of the sample to be studied by referring to the most recent sample or samples of the target sample. Moreover, the neighboring point algorithm only associates with adjacent samples during the execution of the category decision process. Since the neighboring point algorithm mainly relies on a limited number of samples, and does not refer to sample attributes at other levels, the neighboring point algorithm will be used for the sample set to be divided with more overlapping fields. More informative (chen, 2018).

As far as the approach algorithm flow is concerned, it is generally divided into the following steps: First, prepare the preprocessed data, select the appropriate data structure to be studied and test the array and set the parameter K. Secondly, with the parameter K as the target object, the nearest neighbor test array is stored according to the priority order of the distance, and then the detected result is also compared with the array. If the detected distance is higher than the maximum distance sample in the sample array, the value needs to be recalculated; if the detected distance is lower than the maximum distance sample in the sample array, the array can be stored in the priority queue. Finally, the stored priority queue is calculated, and then the error is calculated to re-adjust the parameter K, and finally the K value with the smallest error rate is taken.

2.2 Convex function optimization problem

Convex function optimization, also called convex optimization or convex optimization, convex minimization, is a subfield of mathematical optimization, mainly used to study the problem of convex function minimization. In general, convex function optimization must be the global optimal value in the calculation of mathematical optimal values. Therefore, this convexity advantage of the convex function makes the powerful tools in the convex function analysis be applied in the optimization problem, such as the sub-derivative.As a special existence optimization problem, the convex function optimization problem is now more and more people turn this optimization problem into the unconstrained convex optimization problem. The convex function optimization problem is an important branch of the optimization problem and has a wide range of applications in signal processing and image restoration. Therefore, it is especially important to design fast and feasible algorithms for convex function optimization problems (liu et al, 2018). In recent years, many excellent algorithms have emerged in the convex function optimization problem, such as the conjugate gradient method, the steepest descent method, and the Newton type method. However, such an algorithm applies the convex function optimization problem to the actual situation and does not guarantee the final effect.

3. A Proximal Point Algorithm for Solving Convex Function Optimization Problems

3.1 Preparatory lemma

Suppose H is a real Hilbert space and the inner product is <.,.>,The corresponding inner product induction function is $\|\cdot\|$. Function..: $H \to (-\infty,+\infty)$ is the lower semi-continuous true convex functional.

The proximity algorithm was first proposed by Martinet and is mainly used to solve convex optimal functions. For a given initial point $x_0 \in H$ and the parameter column $\{r_n\}$,the specific iteration sequence is obtained by:

$$x_{n+1} = \arg\min_{x \in H} \left[f(x) + \frac{1}{2rn} \|x - x_n\|^2 \right], n = 1, 2, \cdots (1)$$

Assuming $\sum_{n=0}^{+\infty} r_n = +\infty$,and f has the smallest point, The sequence $\{x_n\}$ generated by equation (1) converges.

In addition to solving convex function optimization, the neighbor algorithm can also be used to solve the zero of the maximal monotone operator, that is, to find $x \in H$, so that $0 \in T(x)$.

f is the lower semi-continuous true convex functional, and the second differential is defined as:

$\partial f(x) = \{z \in H : f(y) \geq f(x) + < z, y - x >, \forall y \in H\}$ (2)

For subdifferential, the essence is the calculation of the maximal monotone operator. To solve the convex function optimization problem, set $x \in H$ as the initial point.

Assuming $f : H \to (-\infty, +\infty)$ is the lower semi-continuous true convex functional, and the resulting sequence always converges to the minimum value P of f .

Compared with the results of previous studies, this paper makes the following optimizations. First, consider the special case of $t_n = 1, e_n = 0$. Second, under the condition of weak convergence, the strong convergence theorem is proved, and the convergence result is more optimized.

To facilitate the proof of the convergence theorem, a format of an equivalent iteration of Equation 2 is given.

The proof process is as follows:

$$x_{n+1} = \arg\min_{x\in H}\left[f(x) + \frac{1}{2rn}\left\| t_n x - x_n \right\|^2 \right] + e_n$$

$$\Leftrightarrow 0 \in \partial f(x_{\mathrm{n+1}} - e_n) + \frac{1}{rn}\left[t_n (x_{\mathrm{n+1}} - e_n) - x_{\mathrm{n}} \right]$$

$$= \frac{1}{t_n} J_{\frac{r_n}{t_n}} x_n$$

$$\Leftrightarrow x_{\mathrm{n+1}} = \frac{1}{t_n} J_{\frac{r_n}{t_n}} x_n + e_n$$

That is, the two equations are equivalent, and the proof ends.

The following two lemmas are used in the proof process.

Lemma 1: Suppose that H is a real Hilbert space and K is a non-empty closed subset of H. P_K is the projection from H to K. For any $x_n \in H$, P_K is the only element of K satisfying $\left\| x - P_K(x) \right\| = \min\left\{ \left\| x - y \right\| : y \in K \right\}$.

For any $x_n \in H$, there exists $< x - P_K(x), P_K(x) - y >\geq 0, \forall y \in K$.

Lemma 2, assuming three non-negative sequences $\{a_n\},\{b_n\},\{c_n\}$, and three sequences satisfy $a_{n+1} \leq (1-b_n)a_n + c_n + o\{b_n\}, \forall n \geq 0$

If $\sum_{n=0}^{\infty} c_n < \infty$,then $\lim_{n\to\infty} a_n = 0$

3.2 Proof process

Prove:As can be seen from $(\partial f)^{-1}(0) \neq 0$,there exists $u \in (\partial f)^{-1}(0)$,so that $J_s u = u$,

$$\left\| x_1 - u \right\| = \left\| \frac{1}{t_0} J_{\frac{r_0}{t_0}} x_0 + e_0 - u \right\| \leq \left\| x_0 - u \right\| + \left\| u \right\| + \left\| e_0 \right\|$$

If $k \in N\{0\}$, there exists $\left\| x_k - u \right\| \leq \left\| x_0 - u \right\| + \left\| u \right\| + \sum_{i=0}^{k-1} \left\| e_i \right\|$,Similar proof

$$\left\| x_{\mathrm{k+1}} - u \right\| = \left\| \frac{1}{t_k} J_{\frac{r_k}{t_k}} x_k + e_k - u \right\| \leq \left\| x_0 - u \right\| + \left\| u \right\| + \sum_{i=0}^{k} e_i$$

Because of $\sum_{k=0}^{+\infty} \left\| e_n \right\| < \infty$, $\{x_n\}$ is bounded, $\left\{ J \frac{r_n}{t_n} x_n \right\}$ is bounded, Setting $r_n \to \infty, t_n \to 1$, $\lim_{n\to\infty} \left\| A \frac{r_n}{t_n} x_n \right\| = 0$ can be obtained.

Next prove: $\lim_{n\to\infty} \sup < 0, P(0) - J \frac{r_n}{t_n} x_n \leq 0$

Where $P : H \to (\partial f)^{-1}(0)$ is a metric projection.

From the basic definition of the upper limit, there is a sequence $\{x_{ni}\} \subset \{x_n\}$, so that B

According to the basic definition of upper limit, there exists a sequence $\{x_{ni}\} \subset \{x_n\}$,make $\lim_{i\to\infty} \sup < P(0), P(0) - J_{\frac{r_{ni}}{t_{n_i}}} x_{n_i} >= \lim_{i\to\infty} \sup < P(0), P(0) - J_{\frac{r_n}{t_n}} x_n >$.

Because $\left\{J_{\frac{r_n}{t_n}} x_n\right\}$ is bound, if it converges to $v \in n$, there exists $v \in (\partial f)^{-1}(0)$. According to the definition of subdifferential, we canget:

$$< f, y >\geq< f, J_{\frac{r_n}{t_n}} x_n > + < A_{\frac{r_n}{t_n}} x_n, y - J_{\frac{r_n}{t_n}} x_n >, \forall y \in H. \left\{J_{\frac{r_n}{t_n}} x_n\right\}$$

For any $\varepsilon > 0$,There exists $P \in N$,when $n \geq p$,there exists $0 \leq \sigma < \varepsilon$,which is $\lim_{n\to\infty} \sigma_n = 0$. It can be inferred that:

$$\|x_{n+1} - P(0)\|^2 = \left\|\frac{1}{t_n} J \frac{r_n}{t_n} x_n + e_n - P(0)\right\|^2$$

$$= \left\|\frac{1}{t_n}(J \frac{r_n}{t_n} x_n - P(0)) - (1 - \frac{1}{t_n})P(0) + e_n\right\|^2$$

$$\leq \frac{1}{t_n}\|(x_n - P(0))\|^2 + \|e_n\|(M + \|e_n\|) + (1 - \frac{1}{t_n})$$

Make $a_n = \|x_n - P(0)\|^2$, $b_n = 1 - \frac{1}{t_n}$, $c_n = \|e_n\|(M + \|e_n\|)$

Available from Lemma 2 , $x_n \to P(0)$.

Because $P(0) \in (\partial f)^{-1}(0)$, according to the definition of subdifferential,

$f(y) \geq f(P(0)) + < 0, y - P(0) >, \forall y \in H$.

which is:

$f(y) \geq f(P(0)), \forall y \in H$.

The proof ends.

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